CONEM 2016
CONGRESSO NACIONAL DE ENGENHARIA MECÂNICA
21 - 25 AGOSTO DE 2016 FORTALEZA - CEARÁ

Modelling, Design and Simulation of a Quadrotor with Tilting Rotors Actuated by a Memory Shape Wire

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Resumo: *This paper deals with a quad rotor hovercraft provided with a mechanism based on a memory shape alloy witch allows tilting the rotors angles. Thus the thrust force vector of each propeller can assume two possible directions: perpendicular to the plane formed by the center of the rotors or tilted inwards. The performed analysis shows that with the second set the vehicle has greater stability, while with the first, there is higher agility. The vehicle's dynamics was studied both analytically and by simulations using MATLAB/Simulink in order to compare the two possible configurations. Based on the analytical study, it is shown that the internal inclination of the rotors, combined with the placement of the vehicle's center of mass below the propellers plane, allows an increase on the maximum torque applied by the controller to establish attitude. Three different scenarios were performed by simulation. In the first scenario, an initial attitude inclination was applied to the hovering Micro Aerial Vehicle (MAV) and the time until stabilization was measured. In the second scenario, with the MAV hovering, it was measured the time it takes to move vertically up to a certain altitude. The MAV with tilted rotors showed better results in the first scenario, wile the conventional configuration showed better performance in the second scenario. The third simulation revealed that the act of tilting the rotors does not change the MAV's position significantly.*
Palavras-chave: *Quad Rotor Hovercraft, Dynamic Model, Memory Shape alloy, Aerial Robotics*

1. Introduction

Recently, due to their huge potential, and growing number of applications, Unmanned Aerial Vehicles (UAVs) have attracted researches attentions (A. Das, 2008). A lot of applications of the UAV of the multirotor type demand its capacity of adapting to different flights situations, like a slow and stable flight (for taking photos or filming, for example) or a cruise flight, where the goal is to quickly reach a position.

Many research groups present demonstrations of how Micro Aerial Vehicles (MAVs) can perform complex maneuvers and tasks. Most of these demonstrations are performed in environments where external sources are used to calculate with accuracy position and attitude of the quadrotors. Using on board measurements, this level of performance is difficult to be achieved, because the information provided by the sensors available for this type of UAVs is ether not reliable, not fast or not accurate enough. Hanoch Efraim (2015) shows that a change in the structure of the quadrotor can produce good hovering dynamics with less dependency on the accuracy of the measurements.

Conventionally, these vehicles are design to have the rotors fixed with respect to the structure (H.Bouadi, 2007 and Pounds, 2007). Mohamed Kara Mohamed (2012) and J. Escareno (2008) present dynamic model of tri-rotor UAVs where the rotors can be tilted to help the control of the vehicle. In order to offer a wider range of control torques, Pau Segui-Gasco (2014) designs and builds a quad rotor hovercraft provided by a mechanism to tilt the propellers. Other mechanism of changing the vehicles structure is presented by Kaan T. Oner (2008), where the UAV tilts its rotors from vertical position to horizontal, in order to take-off/landing vertically, like a helicopter, and fly horizontally, like airplane.

The investigation here proposed, is to analyze, simulate and design a quad rotor hovercraft that can tilt the rotors inwards, making it possible to choose a dynamic configuration that better adapts to the flight regime demanded. The mechanism designed here does not change completely the dynamics of the vehicle, as Kaan T. Oner (2008) does, and is simpler and lighter than the one presented by Pau Segui-Gasco (2014).

In order to tilt the rotors, it is used a NiTi memory shape wire. As current passes through the wire, its temperature increases changing the material phase from detwenned martensite to austenite, decreasing its length. In other hand, when there is no current through the wire, its temperature decreases, and it changes from austenite to detweened martensite when a small stress is applied, increasing its length (Lagoudas,2008). **?** presents some applications of smart material to counter aeroelastic and vibration effects in helicopters and fixed wing aircraft.

The contributions of this work are:

1. Dynamic model of a Quad Tilt Rotor Hovercraft(QTRH).
2. Design of a MAV structure using memory shape alloy to change the dynamics of the quadrotor in flight.
3. Show by simulation the advantages of the proposed design.

This paper is organized in the following sequence: Section 2 defines the forces and torques each rotor produces on the hovercraft adding aerodynamics effects to the model. In this first analysis, some of the drag models for variation of airspeed presented by Moses Bangura (2012) were not considered, the focus was on the aerodynamic effect presented both by Moses Bangura (2012) and Hanoch Efraim (2015), which causes an increase on the lift of each rotor, especially important for stability when the rotors are tilted. The dynamic model considered here also includes the gyroscopic torque generated by the rotation of each propeller combined with the body's change of attitude. With all the torques and forces defined, sections 2.6 and 2.7 model the rotational and translational motions of the vehicle. Lastly, section 2.8 proposes a dynamic model for the tilting actuator using memory shape alloy. Section 3 shows the control laws used for position and attitude control. Section 4.1 choses a combination of parameters that could increase the MAV stability when the rotor inclination angle was maximum (defined as 30 degrees in this work). With the dimensional parameters selected by an analytical study, and respecting constrains of the smart material actuator, section 4.2 shows a design of a structure that is able to change the tilt of the rotor using this wire of shape memory alloy. Section 5 presents simulations that proof greater agility of one configuration and more stability of the other. A third simulation certifies that the act of changing from one configuration to another does not causes a great loss in positioning accuracy.

2. Dynamic Model

2.1 Preliminary Definitions and Notations

2.1.1 Coordinate System

Let the following Cartesian coordinate systems (CCS) be assumed in this work. Body CCS: $S_B \triangleq \{\mathrm{X_B}, \mathrm{Y_B}, \mathrm{Z_B}\}$ fixed to the structure and centered at the body's center of mass (CM). Ground CCS: $S_\mathrm{G} \triangleq \{\mathrm{X_G}, \mathrm{Y_G}, \mathrm{Z_G}\}$, fixed to the ground and centered at a reference O.

2.1.2 Quadrotor

The UAV studied in this paper is a quad rotor hovercraft in an X configuration (the rotors form 45 degrees with S_B on the $X_B - Y_B$ plane) and with the four propeller tilted inwards with the same angle α (Figure 1). The distance between the plane formed by the center of the propellers and center of gravity (CG) of the QTRH is h, the size of the MAV arm is l, f_i and τ_i with $i = 1, 2, 3, 4$ are the force and torque applied by each propeller on the structure.

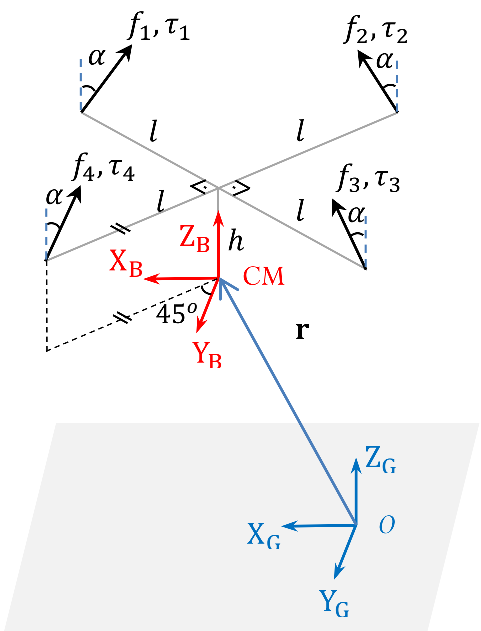

Figura 1: Schematic structure of the Quad Tilt Rotors Hovercraft.

2.2 Rotor Model

Each rotor creates a force f and torque τ in the quadrotor proportional to the squared angular velocity ω of the propeller. Eq.(1) and Eq.(2) below present this result.

$$f_i = k_f \omega_i^2, \tag{1}$$

$$\tau_i = k_\tau \omega_i^2, \tag{2}$$

where k_f and k_τ are coefficients that depend on the air density, geometry of the blade, the attack angle and flow regime. These values were experimentally obtained by Moses Bangura (2012).

The relation between the angular velocity command $\bar{\omega}$ and the actual angular velocity ω in a brushless DC motor can be modeled as a first order system as in Eq.(3).

$$\frac{\omega(s)}{\bar{\omega}(s)} = \frac{k_m}{(\tau_m s + 1)}, \tag{3}$$

where k_m is the DC gain and τ_m is the time constant. Both can be obtained experimentally, for the DC motor.

2.3 Control Force and Torque

It can be shown that the relationship between the force and torque that each rotor provides to the QTRH and the thrust force F^c and the resulting torque $\mathbf{T}^c$ at the quad rotor is given by Eq.(4) and Eq.(5) (Robert Mahony, 2012):

$$\begin{bmatrix} F^c \\ \mathbf{T}^c \end{bmatrix} = \mathbf{\Gamma f}, \tag{4}$$

with $\mathbf{f} \triangleq \begin{bmatrix} f_1 & f_2 & f_3 & f_4 \end{bmatrix}^{\mathrm{T}}$ and

$$\mathbf{\Gamma} \triangleq \sqrt{2}/2 \begin{bmatrix} c\sqrt{2} & c\sqrt{2} & c\sqrt{2} & c\sqrt{2} \\ -lc-ks-hs & -lc-ks-hs & lc+ks+hs & lc+ks+hs \\ -lc+ks-hs & lc-ks+hs & lc-ks+hs & -lc+ks+hs \\ kc & -kc & kc & kc \end{bmatrix}, \tag{5}$$

where $c = \cos(\alpha)$, $s = \sin(\alpha)$ and $k = k_\tau / k_f$. The superscript c stands for *control* and is used to distinguish from other sources of force and torque that will be presented soon.

On the other hand, by inverting Eq.(4), one can compute the command $\bar{\mathbf{f}} \triangleq \begin{bmatrix} \bar{f}_1 & \bar{f}_2 & \bar{f}_3 & \bar{f}_4 \end{bmatrix}^{\mathrm{T}}$ as

$$\bar{\mathbf{f}} = \mathbf{\Xi} \begin{bmatrix} \bar{F}^c \\ \bar{\mathbf{T}}^c \end{bmatrix}, \tag{6}$$

where $\mathbf{\Xi} \triangleq \Gamma^{-1}$, $\bar{F}^c$ is the thrust magnitude command, and $\bar{\mathbf{T}}^c$ is the control torque command.

2.4 Aerodynamic Effect

As it is presented by Hanoch Efraim (2015) , the aerodynamic effect here included is the one induced by the linear velocity combined with the inclination of the rotors. If the airflow through the propeller is increased, the thrust in this propeller is increased. In a hovercraft without the tilted rotor, the lift change cause by this phenomenon is equal to all of the rotors, then it does not generate any torque in the UAV. In a QTRH, however, when the quadrotor is moving sideways, the lift variation is different in the rotors, since the angle between the rotor and the airflow is different. For example, in Figure 2, the vehicle is moving to the right, and, since the rotors are tilted inwards, the airflow into the right rotor is increased and the airflow into the left rotor is decreased, therefor arises a torque acting in the system trying to turn the QTRH to the other side. The lift variation, Δf_i, is given by Eq.(10):

$$\Delta f_i = \frac{1}{4} c M_{cl} \rho_{air} \langle \mathbf{V}_{\mathrm{G}}, \mathbf{P}_{\mathrm{G,i}} \rangle \omega_i, \tag{7}$$

for $i = 1, 2, 3, 4$, where c is the cord of the blade, $M_{cl} = dC_l/daa$ ($C_l$is the lift coefficient and aa is the angle of attack) assumed to be constant for small angles of attack, ρ_{air} is the air density, $\mathbf{V_G}$ is the velocity of the quadrotor in S_G and $\mathbf{P_{G,i}}$ are the vectors normal to each rotor disk in S_G.

$$\mathbf{P}_{\mathrm{G,i}} = \left(\mathbf{D}_{\mathrm{B}}^{\mathrm{B/G}}\right)^{-1} \mathbf{P}_{\mathrm{B,i}}, \tag{8}$$

where $\mathbf{D}_{\mathrm{B}}^{\mathrm{B/G}}$ is the rotational matrix on the body reference frame from S_{G} to S_{B} and $\mathbf{P}_{\mathrm{G,i}}$are the vectors normal to each rotor disk on the body reference frame:

$$\mathbf{P}_{\mathrm{B,1}} = \begin{bmatrix} -\frac{\sqrt{2}}{2}\sin\alpha & \frac{\sqrt{2}}{2}\sin\alpha & \cos\alpha \end{bmatrix}^T, \tag{9}$$

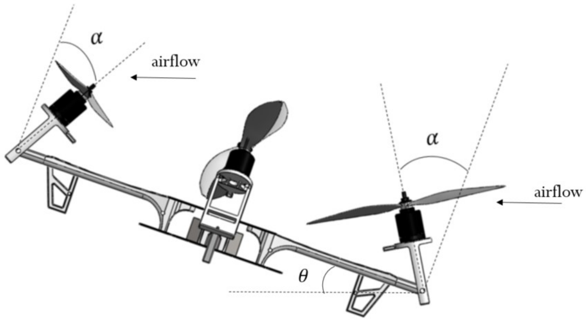

Figura 2: 2D visualization of a QTRH moving to the right (airflow to the left with respect to the hovercraft).

$$\mathbf{P}_{\mathrm{B},2} = \left[\begin{array}{ccc} \frac{\sqrt{2}}{2}\sin\alpha & \frac{\sqrt{2}}{2}\sin\alpha & \cos\alpha \end{array} \right]^T, \tag{10}$$

$$\mathbf{P}_{\mathrm{B},3} = \left[\begin{array}{ccc} \frac{\sqrt{2}}{2}\sin\alpha & -\frac{\sqrt{2}}{2}\sin\alpha & \cos\alpha \end{array} \right]^T, \tag{11}$$

$$\mathbf{P}_{\mathrm{B},4} = \left[\begin{array}{ccc} -\frac{\sqrt{2}}{2}\sin\alpha & -\frac{\sqrt{2}}{2}\sin\alpha & \cos\alpha \end{array} \right]^T. \tag{12}$$

This torque generates a damping component on the quadrotor system. This is desired when we want the UAV to be stable in a position. If a wind perturbation acts over the quadrotor and tilts it sideways, as soon as it starts moving to that side, if the rotors are tilted, this aerodynamic torque will force the quadrotor to tilt to the other side, even before the control system starts acting, increasing the angular velocity of the rotors in one side and decreasing in the other side. Tilting the rotors is especially useful in two scenarios: first if the UAVs is controlled by a human operator, second if the velocity measurement is unavailable or corrupted by noise and delays (Hanoch Efraim, citeyearEfraim15).

2.5 Gyroscopic Torque

As consequence of the combination of the rotation of the blade and the quadrotor body, a torque appears in the system (Hughes (1986)), the gyroscopic torque

$$\mathbf{T}^{gyro} = I \sum_{i=1}^{4} \mathbf{\Omega}_{\mathrm{B}}^{\mathrm{B/R}} \boldsymbol{\omega}_i, \tag{13}$$

where $\boldsymbol{\omega}_i$ is the vector angular velocity of each rotor in the body reference frame S_B, I is the rotational inertia of the propeller w.r.t. its axis of rotation and $\mathbf{\Omega}_B^{B/R}$ is the angular velocity of the body in S_B.

$$\boldsymbol{\omega}_i = (-1)^{i+1}\, \mathbf{P}_{\mathrm{B,i}} \omega_i, \tag{14}$$

for $i = 1, 2, 3, 4$.

2.6 Rotational motion

Representing the attitude using the Euler angles $\boldsymbol{a} \triangleq \left[\begin{array}{ccc} \phi & \theta & \psi \end{array} \right]^{\mathrm{T}}$ in the rotation sequence 1-2-3, we have the following rotational kinematics equations:

$$\dot{\boldsymbol{a}} = \mathcal{A}\mathbf{\Omega} \tag{15}$$

where $\mathbf{\Omega} \triangleq [\Omega_x\ \Omega_y\ \Omega_z]^{\mathrm{T}}$ is the S_{B} representation of the vehicle's angular velocity w.r.t. S_{G} and

$$\mathcal{A} \triangleq \begin{bmatrix} \cos\psi/\cos\theta & -\sin\phi/\cos\theta & 0 \\ \sin\psi & \cos\psi & 0 \\ -\cos\psi\sin\theta/\cos\theta & \sin\psi\sin\theta/\cos\theta & 1 \end{bmatrix} \tag{16}$$

Assume that the vechile has a rigid structure and S_{G} is an inertial frame. Considering the existence of gyroscopic (due to the rotors) and disturbance torques, and using the Newton-Euler formulation, one can model the rotational dynamics of the quadrotor vehicle by

$$\dot{\mathbf{\Omega}} = \mathbf{J}^{-1}(\mathbf{J}\mathbf{\Omega}) \times \mathbf{\Omega} + \mathbf{J}^{-1}\mathbf{T}^{gyro} + \mathbf{J}^{-1}\mathbf{T}^{c} + \mathbf{J}^{-1}\mathbf{T}^{d}, \tag{17}$$

where $\mathbf{T}^c$,$\mathbf{T}^{gyro}$ and $\mathbf{T}^d$ are the S_{B} representations of the resultant control torque, gyroscopic torque and disturbance torque, respectively and J is the body inertia matrix. Consider that the vehicle has a symmetric structure with known mass m and inertia matrix in S_{B}

$$\mathbf{J} = \begin{bmatrix} J_x & 0 & 0 \\ 0 & J_y & 0 \\ 0 & 0 & J_z \end{bmatrix}. \tag{18}$$

2.7 Translational Motion

Consider that the Earth is flat and the gravitational acceleration g is constant anywhere the vehicle is supposed to operate. Therefore, the S_{G} representation of the gravitational acceleration vector is given by

$$\mathbf{g} = \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix}. \tag{19}$$

Denote the S_{G} representations of the position and velocity of S_{B} w.r.t. S_{G} by $\mathbf{r} = [r_x\ r_y\ r_z]^{\mathrm{T}}$ and $\mathbf{v} = [v_x\ v_y\ v_z]^{\mathrm{T}}$, respectively. Therefore we can describe the vehicle's translational kinematics and dynamics by

$$\dot{\mathbf{r}} = \mathbf{v}, \tag{20}$$

$$\dot{\mathbf{v}} = \frac{F^c}{m}\mathbf{n} + \mathbf{g} + \frac{1}{m}\mathbf{F}^d, \tag{21}$$

where $\mathbf{n} \triangleq [\ \sin\phi \quad -\sin\phi\cos\theta \quad \cos\phi\sin\theta\ ]^{\mathrm{T}}$ is the S_{G} representation of the unit vector perpendicular to the plane of the rotors, $\mathbf{F}^d$ is the S_{G} representation of the disturbance force, m is the vehicle's total mass, and F^c is the magnitude of the resultant control thrust.Davi Antonio dos Santos (2013)

2.8 Smart Material Actuator Model

Due to the complexity of the actuation system for tilting the rotors using memory shape wire, it was modeled as a first order system:

$$\frac{\alpha(s)}{\bar{\alpha}(s)} = \frac{k_\alpha}{(\tau_\alpha.s + 1)}, \tag{22}$$

where, k_α and tau_α are respectively the gain and the constant of time and must be experimentally estimated.

3. Control Law

The control is divided in two parts. A inner loop to control attitude and a outer loop to control position just like it is done by Davi Antonio dos Santos (2013) and shown in the figure below.

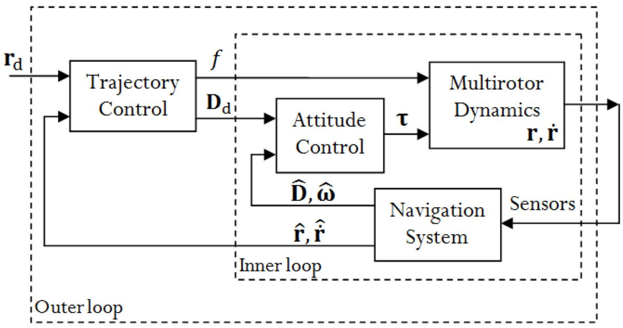

Figura 3: Schematic structure of a MAV control system.

3.1 Attitude Control

Considering that the vehicle moves only around one S_B axis per time and taking in considerations the dynamic model, the rotational motion can be simplified to three double integrators decoupled that, written in state space form become:

$$\dot{\boldsymbol{x}}_i = \boldsymbol{A}\boldsymbol{x}_i + \boldsymbol{B}_i\left(\mathbf{T}_i^c + \mathbf{T}_i^d\right), \tag{23}$$

where $i = x, y, z$. The state vector is $\boldsymbol{x}_i = [x_{i1}\ x_{i2}]^T$, where $x_{i1} \triangleq \boldsymbol{a}_i$ and $x_{i2} \triangleq \dot{\boldsymbol{a}}_i$, and:

$$\boldsymbol{A} = \begin{bmatrix} 0\,1 \\ 0\,0 \end{bmatrix}, \boldsymbol{B}_i = \begin{bmatrix} 0 \\ 1/J_i \end{bmatrix}. \tag{24}$$

The following state feedback control law was chosen:

$$\mathbf{T}_i^c = \mathrm{sat}_{[-\mathrm{T}_{\max,i},\mathrm{T}_{\max,i}]}(-\boldsymbol{K_i}\boldsymbol{x}_{\mathrm{i}} + \mathrm{N}_{\mathrm{i}}\bar{\boldsymbol{a}}_{\mathrm{i}}), \tag{25}$$

where $\boldsymbol{K}_i = [K_{i1}\ K_{i2}]^T \in \mathbb{R}^{1\mathrm{x}2}$ and $N_i \in \mathbb{R}$. For having a zero steady state error N_i need to be equal K_{i1} (for $\boldsymbol{T^d}_i = 0$). By placing the closed loop poles at $\lambda_i \in \mathbb{R}_-^*$, $K_{i1} = J_i(\lambda_i)^2$ and $K_{i2} = -2J_i(\lambda_i)$ can be determined.

Due to the maximum thrust force each rotor can support, there must be a saturation function to limit the torque. These topic is explored in the design section.

3.2 Position Control

Assume that the control thrust force $\bar{F}$ is equal the norm of the desired thrust force $||\vec{\mathbf{F}}^{c}||$ and that the desired attitude matrix is equal the actual attitude $\left(\bar{\mathbf{D}}^{\mathrm{B/R}} = \mathbf{D}^{\mathrm{B/R}}\right)$. Definitions:

1. The unitary vector perpendicular to the rotors plane can be written as $\mathbf{n} = \mathbf{F}^{c}_{\mathrm{G}}/F$.
2. $\varphi \triangleq \cos^{-1}\left(\mathbf{n}^{T}\boldsymbol{e}_3\right)$ the inclination angle of the rotors plane.
3. Control error is given by $\tilde{\mathbf{r}} = \mathbf{r}^{\mathrm{B/G}}_{\mathrm{G}} - \bar{\mathbf{r}}^{\mathrm{B/G}}_{\mathrm{G}}$, where $\bar{\mathbf{r}}^{\mathrm{B/G}}_{\mathrm{G}}$ is the position command.

Command of $\mathbf{n}$ is given by $\bar{\mathbf{n}}$ and command of φ is given by $\bar{\varphi}$. The problem consists in designing a control law to bring $\tilde{r}$ to zero, respecting the following restrictions: $F_{min} \leq \bar{F} \leq F_{max}$ and $\bar{\varphi} \leq \varphi_{max}$. As it is done by Davi Antonio dos Santos (2013), this problem is divided in: altitude control and horizontal position control.

3.2.1 Altitude Control

From translational motion in z axis:

$$\ddot{r}_z = \frac{\bar{F}}{m}\bar{n}_z + \frac{1}{m}F^{d}_{z} - g \tag{26}$$

where F^{d}_{z} is the z component of the disturbance force. The following state feedback control law was chosen:

$$\bar{F} = \mathrm{sat}_{[\mathrm{F_{min}},\mathrm{F_{max}}]}\left(\frac{m}{\bar{n}_z}\left(g - k_1(r_z - \bar{r}_z) - k_2 v_z\right)\right) \tag{27}$$

k_1 and k_2 were adjusted for the system throught pole placement.

3.2.2 Horizontal Position Control

The translational motion in the plane $\mathrm{X_G} - \mathrm{Y_G}$ is given by:

$$\ddot{\mathbf{r}}_{xy} = \frac{\bar{F}}{m}\bar{\mathbf{n}}_{xy} + \frac{1}{m}\mathbf{F}^{p}_{xy}, \tag{28}$$

where $\bar{\mathbf{n}}_{xy} = [\bar{n}_x\ \bar{n}_y]^T$, $\mathbf{r}_{xy} = [r_x\ r_y]^T$ and $\mathbf{F}^{p}_{xy} = \left[F^{p}_{x}\ F^{p}_{y}\right]$. To control horizontal position $\mathbf{r}_{xy}$ the following controller was used:

$$\bar{\boldsymbol{n}}_{\boldsymbol{xy}} = \begin{cases} \frac{\gamma_{xy}}{||\gamma_{xy}||}\sin\varphi_{max}, & \text{if } ||\gamma_{xy}|| > \sin\varphi_{max} \\ \gamma_{xy}, & \text{otherwise} \end{cases}, \tag{29}$$

where

$$\gamma \triangleq \frac{m}{\bar{F}}(-k_3(\boldsymbol{r}_{xy} - \bar{\boldsymbol{r}}_{xy}) - k_4\boldsymbol{v}_x y) \tag{30}$$

k_3 and k_4 were also adjusted through pole placement.

4. Structure Design

Besides the bonus of having a damping factor intrinsic to the system, added by the Aerodynamic Effect discussed in section 2.5, the great advantage of tilting the rotors is that there is a possibility of raising the maximum torque of control in the QTRH without raising the DC motors capacity. The maximum control torque that can be applied to a usual Quadrotor in X (rotors not tilted) is given by:

$$\mathbf{T}_{max-normal} = \left[\ \sqrt{2}l\ \ \sqrt{2}l\ \ 2.k\ \right]^T(\zeta_{max} - \zeta_{min}), \tag{31}$$

where

$$\zeta_{max} \triangleq \min\left\{\bar{f}_{min}, (1/2)\left(\bar{F} - \bar{f}_{max}\right)\right\}, \tag{32}$$

$$\zeta_{min} \triangleq \max\left\{\bar{f}_{max}, (1/2)\left(\bar{F} - \bar{f}_{min}\right)\right\}, \tag{33}$$

$\bar{f}_{min}$ and $\bar{f}_{mmax}$ are respectively the minimum and maximum forces of each rotor and $\bar{F}$ is the thrust force necessary to support the hovercraft weight. For a QTRH, the maximum torque is defined by the Eq.(29) below:

$$\mathbf{T}_{max-tilted} = \left[\ \sqrt{2}(l\cos\alpha + h\sin\alpha)\ \ \sqrt{2}(l\cos\alpha + h\sin\alpha)\ \ 2kcos\alpha\ \right]^T(\zeta_{max} - \zeta_{min}). \tag{34}$$

With this equations it can be observed that a combination of l,α and h can increase the maximum torque in directions x and y when tilting the rotors. For a fixed tilting angle of 30 degrees:

$$\frac{T_{max-tilted}(x)}{T_{max-normal}(x)} = \frac{\sqrt{3}}{2} + \frac{h}{2l} \tag{35}$$

For $l = 0.183m$ and $h = 0.1039m$ it is possible to increase in 15% the maximum torque tilting the rotors inwards.

With the design parameters fixed (l, h and α) a quadrotor was designed to satisfy these parameters. Other parameters that was taken in consideration to design the structure arm was the size of the shape memory wire, since the maximum variation of the wire length is $\Delta l/l = 4\%$. In Figures 4 and 5 the QTRH design is shown. The structure was designed in such a way that the rotors are allowed to vary between 0 and 30 degrees with respect to the $\mathrm{Z_B}$ axis in S_B. When the rotors are desired to be tilted, the memory shape wires need to be loose, then a spring will bring the rotors to the configuration on the left in Figure 4. When it is desired to have strait rotors, current will pass through the memory shape wires, inducing them to change their material properties, decreasing their length. As this happens the traction force on the wire will generate more torque on the structure part that holds the rotor than the elastic force and will bring the rotor to the configuration on the right in Figure 4.

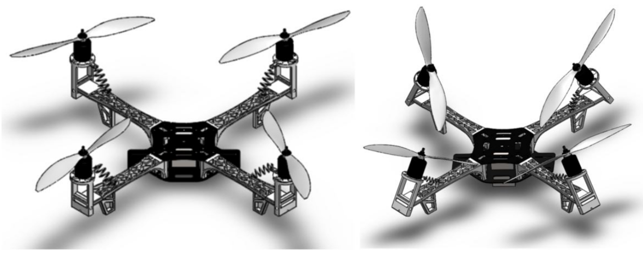

Figura 4: Isometric view of the QTRH with the rotors in the straight postion (left) and in the tilted position (right).

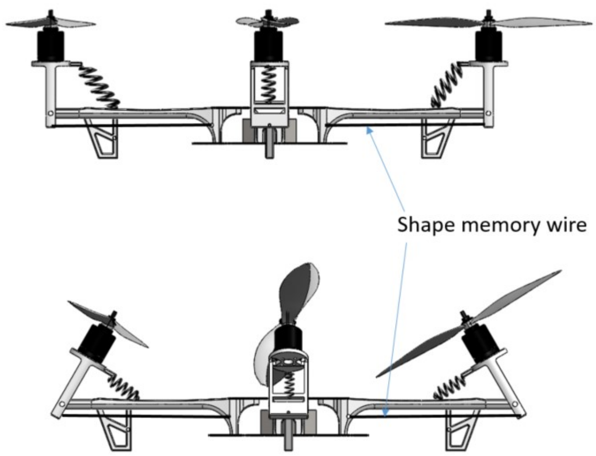

Figura 5: Side view of the QTRH with the rotors in the straight and tilted position.

5. Simulation

A Simulink model was created to simulate the QTRH, and the outer (position) and inner (attitude) control loops. With this model, it was studied the performance of the two configurations of the hovercraft (when the rotors are tilted 0 or 30 degrees) through two tasks.

5.1 Task 1: Z position

In this first task, the UAV, needs to reach position $[2\ 2\ 2]^T$ leaving from position $[2\ 2\ -20]^T$. So moving only in the z axis. The results from the simulation are shown in Figure 6 below. From these graphics, it can be noticed that the for the position error in the z axis be less than 0.1 meters, the tilted rotor quadrotor takes 6.40 seconds, while the strait quadrotor takes 5.83 seconds.

5.2 Task 2: Initial Inclination

In this task, the UAV, needs to stay in the position $[2\ 2\ 2]^T$ leaving from the same position but rotated of 75 degrees in the x axis.

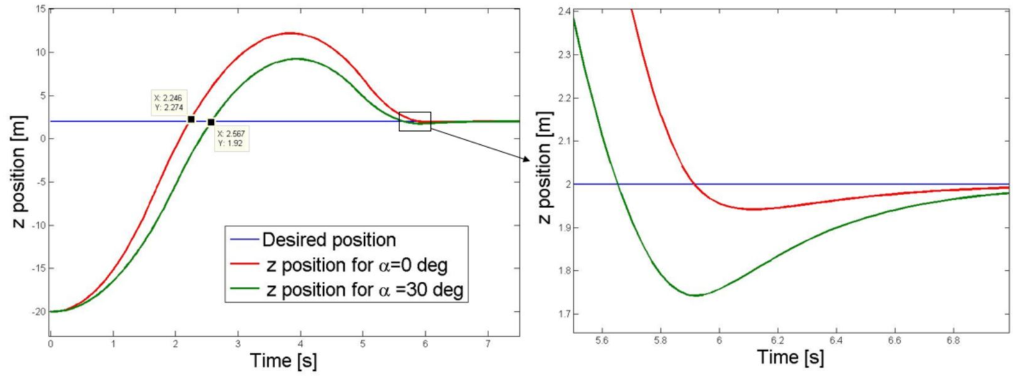

Figura 6: Results of the simulation of task 1.

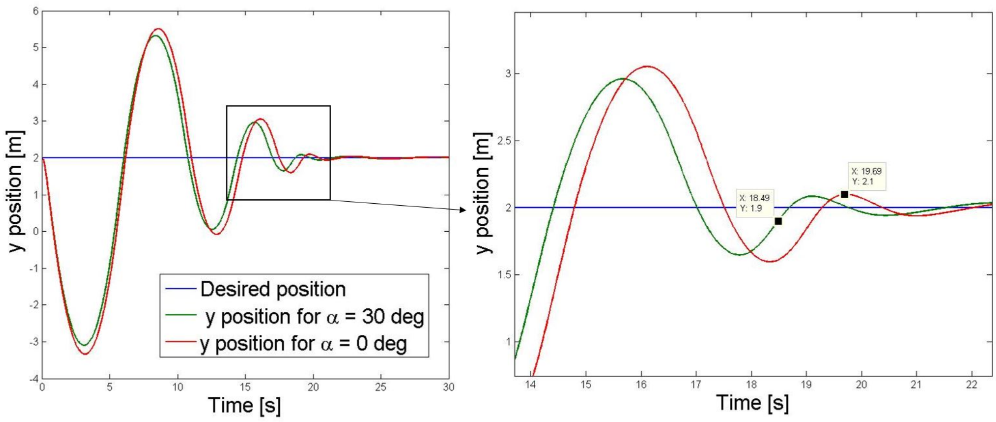

Figura 7: Position y from the results of the simulation of task 2.

Now it can be observed that, for the position error in the y axis be less than 0.1 meters, the tilted rotor quadrotor takes 18.49 seconds, while the quadrotor with straight rotors takes $19{,}69$ seconds.

In the attitude, for ϕ to be less than 1 degree, the QTRH takes 20.55 seconds while the quad rotor with straight rotors takes 21.17 to stabilize the position.

5.3 Task 3: Tilting rotor on hovering

In this task, the rotors inclination is changed from 0 to 30 degrees at 0 seconds and then from 30 degrees to 0 at 8 seconds. The MAV starts hovering in the position $[2\ 2\ 2]^T$ and has the goal to stay in the same positon.

As soon as the command to tilt the rotors is given to the shape memory alloy, the control law understands that the rotors go instantaneously to the final position and increases the rotation of the propellers, in order to keep the desired force, but, since the inclination angle of the rotors follows a first order dynamic, the thrust force becomes higher than it needed to be to hold the MAV in the position. As a consequence of this delay, the z position of the MAV increases. Analogously, when the command to bring the rotors back to the straight position is give, the control law understands that the rotors become instantaneously strait, and decreases the rotation of the propeller,but the rotors are still going to the final position, so the thrust force decreases and the UAV loses altitude.

In this task, there is change on the MAV's position only in the z axis, the others stay constant. The altitude variation wile tilting the rotors is 0.045 meters, and wile bringing the rotors back to strait position is 0.029 meters.

6. Conclusions

With the result from the simulation of task 1, it can be concluded that a quadrotor with straight rotor performs the task with more success than a QTRH, even though the overshoot is higher, it reaches the final position in 10% less time. This happens because the maximum thrust force is multiplied by $\cos\alpha$ when tilting the rotor an α angle. In other hand, the QTRH performs the second task with more precision than the normal, reaching the position 6% faster. It happens because the quadrotor with tilted rotors was designed to have 15% more maximum torque than the quadcopter with straight rotors. In simulation 3, it is presented that changing the rotors inclination in flight does not change significantly the position of the MAV.

From these results, it can be concluded that this design of UAV is able to change between a configuration of more stability to another allowing more agility, during the flight, with little loss in accuracy of positioning.

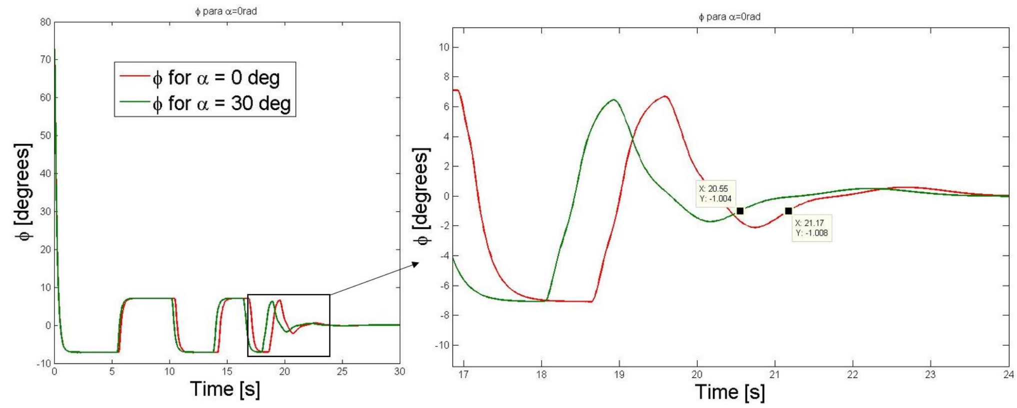

Figura 8: Inclination ϕ from the results of simulation of task 2.

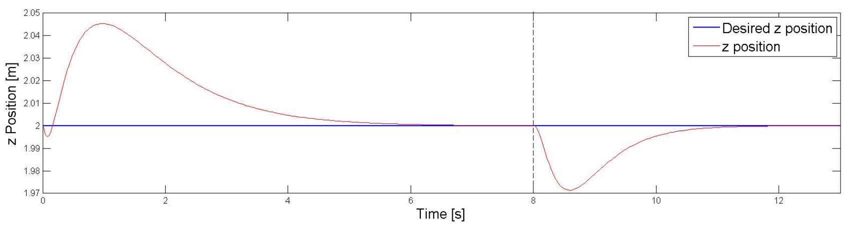

Figura 9: Z position of the MAV wile the rotors are being tilted from 0 to 30 degrees at 0 seconds and from 30 degrees to 0 at 8 seconds.

7. Future Works

Future works will include three phases. First, application of different control techniques to this model and analyze which one better suits the problem. Secondly, construction of the vehicle and test of the tilting mechanism in flight. Lastly an experimental verification of the results presented here.

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